sity; Θ, model temperature; α, regularization parameter; δ_{L_2}, integral error of experimental temperature. Indices: max, maximum value; pul, pulse.

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HEAT CONDUCTION IN THE QUASISTEADY HEATING OF MATERIALS

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In determining the thermal conductivity of ablative thermally protective materials (APM) by solving the inverse (coefficient) problem of heat conduction (ICP) and calculating the temperature fields in thermally protective coatings based on such materials, investigators encounter several difficulties. The main difficulty relates to the indeterminateness of certain parameters and characteristics in the mathematical model of heat transfer for these materials under conditions of intensive unidirectional heating. Foremost among these parameters and characteristics are the temperature dependences of the density and volumetric specific heat of the given material at high temperatures and the parameters of decomposition of the binder of the APM.

Here, we use the example of the quasisteady regime of heating of APM (steady-state ablation) to analyze the effect of these factors on the thermal conductivity and temperature field of the APM.

With allowance for the heat sinks, we will represent the solution of the direct heat conduction problem (DCP) for quasisteady heating of an APM presented in [1] in the form

$$x = \frac{1}{V}\int_{T}^{T_w}\frac{\lambda(T)\,dT}{\int_{T_0}^{T} C_p(T)\,dT - \Delta H^*\int_{T_0}^{T}\frac{\partial\rho}{\partial T}\,dT - c_g\int_{T_0}^{T}\int_{T_0}^{T}\frac{\partial\rho}{\partial T}\,dT\,dT}. \quad (1)$$

In the heating regime being considered here, the capacity of the internal heat sinks has the form

$$q_v = V\left(\Delta H^*\frac{\partial\rho}{\partial T} - c_g\int_{T_0}^{T}\frac{\partial\rho}{\partial T}\,dT\right)\frac{dT}{dx}.$$

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Using (1) to solve the ICP, we obtain the following method of determining the thermal conductivity of an APM. Using thermocouples, we determine the change in temperature over time in some cross section of the test material during quasisteady heating. We divide $T(\tau)$ into N intervals. Assuming that the thermophysical characteristics (TPC) of the material are piecewise-constant functions of temperature and performing some simple transformations of Eq. (1), we obtain

$$\hat{\lambda}_m(\bar{T}_m) = \frac{V^2\Delta\tau_m C_{vm}}{\ln\left(1 + \dfrac{\Delta T C_{vm}}{\Delta T\sum\limits_{i=1}^{m-1} C_{vi}\sigma_i - \Delta H^* \int\limits_{T_0}^{T_m} \dfrac{\partial\rho}{\partial T}\,dT - c_g \int\limits_{T_0}^{T_m}\int\limits_{T_0}^{T} \dfrac{\partial\rho}{\partial T}\,dT\,dT}\right)}. \tag{2}$$

In the case of a quadratic temperature dependence of the density of the APM in the pyrolysis zone, the theoretical relations for determining the thermal conductivity of the APM have the form

$$\hat{\lambda}(\bar{T}_m) = \begin{cases} \dfrac{V^2\Delta\tau_m C_{vm}}{\ln\left(1 + \dfrac{C_{vm}}{\sum\limits_{i=1}^{m-1} C_{vi}\sigma_i}\right)}, & T_0 \leqslant T < T_b, \\ \dfrac{V^2\Delta\tau_m C_{vm}}{\ln\left(1 + \dfrac{\Delta T C_{vm}}{\Delta T\sum\limits_{i=1}^{m-1} C_{vi}\sigma_i + \rho_0\varphi\Gamma\left(\dfrac{T_m - T_b}{T_e - T_b}\right)^2 [\Delta H^* + (T_m - T_b)\,c_g/3]}\right)}, & T_b \leqslant T \leqslant T_e, \\ \dfrac{V^2\Delta\tau_m C_{vm}}{\ln\left(1 + \dfrac{\Delta T C_{vm}}{\Delta T\sum\limits_{i=1}^{m-1} C_{vi}\sigma_i + \rho_0\varphi\Gamma\,[\Delta H^* + (3T_m - 2T_e - T_b)\,c_g/3]}\right)}, & T_e < T \leqslant T_w. \end{cases} \tag{3}$$

To realize the above-described method, it is necessary to know the temperature at which pyrolysis of the polymer binder of the APM begins and ends. These temperatures depend appreciably on the heating rate [2] and are distinct, unlike, for example, the melting point of crystalline substances. Thus, below we propose a method of evaluating these temperatures under conditions of intensive unidirectional heating. The method involves the use of the same experimental data as is used in determining $\lambda(T)$.

We use the first relation of (3) to determine $\lambda(T)$ throughout the range T_0-T_w and we use the extrema of this function to evaluate the temperatures at which pyrolysis of the binder beings and ends (curve 4 in Fig. 1). The first extremum—maximum of $\lambda(T)$ is connected with the onset of the formation of secondary porosity in the material as a result of thermal decomposition of the binder. At the end of this process, the porosity of the material (coke) stabilizes, and heat conduction begins to increase (second extremum). These temperatures can be similarly determined in other heating regimes with the use of different methods of solving the ICP.

Equations (1)-(3) were used to analyze the effect of different factors on the thermal conductivity and temperature field of an APM. Here, we made use of the decomposition parameters of the epoxy binder, along with the temperature dependences of the thermal conductivity and volumetric specific heat of the epoxy glass-fiber-reinforced plastic obtained in [3].

It follows from Eqs. (1) and (2) that both the temperature field and the thermal conductivity of the APM depend on the integral of the function $\partial\rho/\partial T = f(T)$. To determine the degree of the effect of different temperature dependences of the density of the material in the binder pyrolysis zone on $T(x)$ and $\lambda(T)$, we performed calculations with Eqs. (1) and (2) using different $\rho(T)$. The latter was obtained from thermogravimetric analysis data for an epoxy glass-plastic (ρ_1) in [3] and the quadratic (ρ_2) and linear (ρ_3) relations for the density of the binder in the pyrolysis zone.

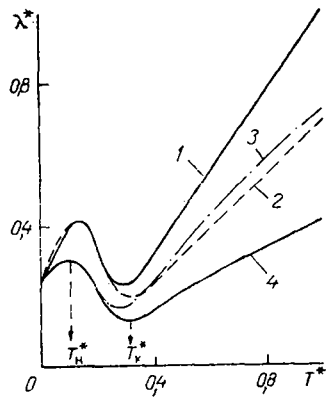

Fig. 1

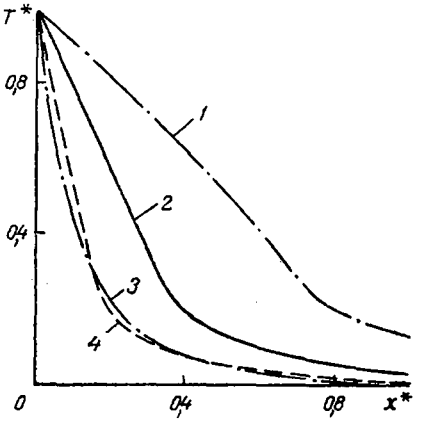

Fig. 2

Fig. 1. Effect of certain factors on the thermal conductivity of glass-fiber-reinforced plastic: 1) initial $\lambda(T)$; 2-4) $\lambda(T)$ established at $q_V \neq 0$, $C_V(T)$; $q_V \neq 0$, $C_V(T) = C_{V0}$; $q_V = 0$, $C_V(T) = C_{V0}$, respectively (2-4 were established from T(x) calculated with $C_V(T)$, the initial $\lambda(T)$, and $\rho_1(T)$). $\lambda^* = \lambda/\lambda_{max}$, $T^* = T/T_W$ (T_W = 1600°C).

Fig. 2. Effect of certain factors on the temperature field in the glass-plastic: T(x) was calculated with the following data: 1) the initial $\lambda(T)$, $C_V(T) = C_{V0}$, $q_V = 0$; 2) the initial $\lambda(T)$, $C_V(T)$, $q_V \neq 0$ (main variant); 3) $\lambda(T) = \lambda_0$, $C_V(T)$, $q_V \neq 0$; 4) $\lambda(T)$ (curve 4, Fig. 1), $C_V(T)$, $q_V \neq 0$, $x^* = x/(6 \cdot 10^{-3}$ m).

The calculations established that the maximum difference from the main variant ($\rho_1(T)$) for the temperature field is roughly +3% with the use of $\rho_2(T)$ and about +6% with $\rho_3(T)$. In all three cases of the temperature dependence of density in the pyrolysis zone, the temperature field was calculated with the same $\lambda(T)$ and $C_V(T)$. To calculate T(x) with $\rho_2(T)$ and $\rho_3(T)$, we first determined the temperatures of the onset and termination of decomposition of the binder by the method described above (extrema on curve 4 in Fig. 1). Here, we used the temperature field of the main variant. These temperatures were determined with zero and −7.8% errors, respectively. The use of the linear and quadratic temperature dependences of density leads to an increase in $\lambda(T)$. The maximum deviations are +14.9 and +6.4%, respectively.

The data presented above indicates that thermogravimetric studies of an APM are not necessary either to calculate the temperature field or to establish the thermal conductivity of the material (by solving the ICP). It is necessary only to determine the temperature corresponding to the onset and termination of binder decomposition. These temperatures are determined using the same experimental data as was used to determine $\lambda(T)$ (i.e., no special experiments need to be performed). Assigning a quadratic temperature dependence for the density of the material, we can calculate the temperature field and thermal conductivity with an accuracy sufficient for engineering purposes.

Although the above analysis was performed for a quasisteady heating regime, it is evidently also valid for other regimes, since it is necessary to integrate the heat conduction equation in any case.

When the thermal conductivity of an APM is determined by solving the ICP, the temperature dependence of the volumetric heat capacity of the material is often unknown. The effect of constancy (at 20°C) of this characteristic on $\lambda(T)$ for the test material (with $q_V \neq 0$) is shown by curve 3 in Fig. 1. The maximum deviation of this curve from the initial $\lambda(T)$ (curve 1 in Fig. 1) is approximately −29%. Allowance for the temperature dependence of the volumetric heat capacity of the material ($q_V \neq 0$) leads to a maximum error of +4%. The $\lambda(T)$ established with $C_V(T)$ lies somewhat higher than the original $\lambda(T)$ throughout the investigated temperature range. However, this difference is small enough so that, except for the temperature range corresponding to pyrolysis of the binder, the established $\lambda(T)$ nearly coincides with curve 1 in Fig. 1.

The greatest deviation of the established $\lambda(T)$ from the initial relation occurs in the case of constancy of volumetric heat capacity and equality of the internal heat sinks to zero (curve 4 in Fig. 1). The maximum deviation is −59.4% at T_w.

The literature data on the heat of decomposition of binders and the mass heat capacity of their gaseous decomposition products has a large scatter. An analysis of the effect of these factors [alternate triviality of ΔH and c_g in (2)] on the established thermal conductivity of the material shows that the established $\lambda(T)$ differs from the initial relation at temperatures greater than the temperature at which decomposition of the binder ends. Meanwhile, triviality of the heat capacity of the binder's gaseous decomposition products has the greatest effect (the maximum deviation is −29%, while it is −6.4% at $\Delta H = 0$). Thus, the error of c_g has the greater effect on $\lambda(T)$ in the solution of the ICP.

In connection with the fact that the temperatures of the onset and termination of pyrolysis of the binder of the glass-plastic were obtained with a small error, we established $\lambda(T)$ with different variants of change in these temperatures by ±100°C (±50% and ±19%, respectively). The maximum deviation (+22.2) of the established $\lambda(T)$ from the initial relation was obtained with a simultaneous reduction in the temperature corresponding to the onset and termination of binder pyrolysis.

If we use Eq. (1) and the results for $\lambda(T)$ shown in Fig. 1 to calculate the temperature fields in an APM with the same values for surface temperature and linear ablation rate, we find the following to be true. When T(x) is calculated with the temperature dependence of the thermal conductivity of the material [established using $C_v(T)$ and $q_v \neq 0$, which nearly agrees with the initial $\lambda(T)$], a constant volumetric heat capacity (at 20°C), and triviality of the internal heat sinks, then there is a significant difference from the temperature field of the main variant (curves 1 and 2 in Fig. 2). Meanwhile, the maximum deviation with respect to temperature is approximately +200%. In the given case, we used the complete mathematical model of heat transfer in an APM to determine the thermal conductivity and we use a simplified model to calculate T(x). A similar result was obtained with the opposite formulation, i.e., when $\lambda(T)$ was determined using the simplified mathematical model of heat transfer (curve 4 in Fig. 1) and T(x) was calculated using this established $\lambda(T)$ and the temperature dependence of volumetric specific heat with allowance for the heat sinks (curve 4 in Fig. 2). In this case, the maximum deviation for temperature from T(x) of the main variant (curve 2 in Fig. 2) is −63%. However, if $\lambda(T)$ is determined from the temperature field of the main variant even with C_{v0} and $q_v = 0$ and if we use this data to calculate T(x), then this temperature field agrees completely with the T(x) of the main variant, i.e., all of the "shortcomings" of the mathematical model of heat transfer in the material are compensated for by the thermal conductivity.

Calculation of the temperature field with a constant thermal conductivity (20°C), the temperature dependence of volumetric heat capacity, and the presence of heat sinks (curve 3 in Fig. 2) also leads to a large deviation from T(x) of the main variant. Failure to allow for the temperature dependence of thermal conductivity when calculating the temperature field in the APM has two consequences: It leads to a sizable error; it leads to a change in the character of T(x) (compare curve 3 with curves 1, 2, and 4 in Fig. 2).

Thus, consistent mathematical models of heat transfer in an APM must be employed when determining the thermal conductivity of this material and when calculating the temperature field from thermal conductivity. Only in this case can a minimal error in the calculation of T(x) be obtained. Although this conclusion follows directly from Eqs. (1) and (2), the above analysis was performed with a specific example to qualitatively show the large errors that can result from inconsistency of the mathematical models of heat transfer in an APM in the solution of the inverse and direct heat conduction problems. A similar situation will prevail in other cases where heat transfer in materials is complicated (such as in translucent materials).

NOTATION

x, coordinate of the movable coordinate system connected with the moving surface of the material; V, linear ablation rate; T, temperature; ρ, density; λ, thermal conductivity; C_v, volumetric heat capacity; $\Delta H^* = \Delta H$; φ, content of binder in the material; ΔH, heat of decomposition of the binder; c_g, heat capacity of the gaseous decomposition products of the binder; Γ, gasification coefficient of the binder; T_0, initial temperature; T_w, temperature

of the surface; ρ_0, density of the initial material; T_b, T_e, temperature of beginning and end of binder decomposition; $\Delta\tau_m = \Delta x_m/V$, time necessary for the temperature in the chosen cross section of the specimen to increase from T_{m-1} to T_m; σ_i, unit function; $\overline{T}_m = (T_m + T_{m-1})/2$, $i = \overline{1, N}$; $m = i + 1$; $\Delta T = T_m - T_{m-1}$.

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IDENTIFICATION OF THE COMBUSTION FRONT OF AN OIL RESEVOIR

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A method is proposed for determining the rate of movement of the combustion zone in an oil resevoir from temperature perturbations of the earth's surface layer.

One of the most promising methods of increasing the yield of heavy and viscous fuel from oil resevoirs is the creation of a combustion zone in the resevoir. Realization of this method involves the creation of a system of igniting wells and operating systems. The combustion front, initially coincident with the surface of the igniting well, moves continuously in the direction of the operating wells at a rate of 5-15 cm a day. The form and velocity of the front depend on several factors, including the heterogeneity of the resevoir, the number and location of the operating wells, the physical properties of the resevoir, the composition, concentration, and rate of extraction of the oil, and the consumption of injected air.

It is necessary to control the combustion process in order to ensure stable resevoir combustion, an optimum shape for the combustion front, and fuller coverage of the height of the resevoir by the front. In order to examine the feasibility of using recorded temperature perturbations of the earth's surface layer caused by a combination front to determine the position of the front, the problem of heat transfer in a rock mass was examined with the following assumptions. The temperature field in the mass is described by the heat conduction equation. Before the beginning of combustion of the resevoir, the temperature field is a function of a single space coordinate z. The z axis is directed along an interior normal to the earth's surface. Boundary conditions of the third kind exist on this surface. In the subsurface layers of the earth ($z > z^*$), the temperature can be assumed to be independent of time. A combustion front develops at the moment of time $\tau = 0$ in the resevoir $z_{in} \le z \le z_{ex}$. The front is a right circular cylinder of finite length $H = z_{ex} - z_{in}$ with its axis parallel to the z axis. The readius of the base of the cylinder is a function of τ: $R = R(\tau)$. During the period of time $0 < \tau < \tau'$, when $R(\tau) < s$, the combustion zone retains the form of a circular cylinder. At $\tau > \tau'$, the zone takes the form of a hollow circular cylinder. The rates v of the change in the radii of the internal r_{in} and external cylinders are the same, i.e., $R - r_{in} = s$. The temperature on the combustion front does not change over time.

The combustion front is identified from the values of excess temperature relative to the unperturbed temperature field in the rock mass. The excess temperature function satisfies the following system of equations:

$$c\rho \frac{\partial t(r, z, \tau)}{\partial \tau} = \frac{\partial}{\partial z}\left(\lambda \frac{\partial t(r, z, \tau)}{\partial z}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda \frac{\partial t(r, z, \tau)}{\partial r}\right); \quad (1)$$

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